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First passage time of nonlinear ship rolling in narrow band non-stationary random seas

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Abstract

The generalized extended stochastic central difference (GESCD) method is applied to study the response statistics and first passage time of nonlinear ship rolling in narrow band stationary and non-stationary random seas. The GESCD method is based on a combination of the extended stochastic central difference method with a statistical linearization technique, modified adaptive time scheme, and time coordinate transformation. The extended stochastic central difference method is, however, an extension of the stochastic central difference method for the determination of the recursive mean square or covariance of responses of systems under narrow band stationary and non-stationary random disturbances. Approximate first passage probabilities of nonlinear systems based on the modified mean rate of various crossings proposed earlier by the first author were determined. It is concluded that the GESCD method is very accurate, simple and efficient to apply compared with Monte Carlo simulation. The proposed method is applicable to cases with large nonlinearities and intensive random excitations. The approximate first passage probabilities of the nonlinear system determined by the proposed approach are very accurate as they are in excellent agreement with those evaluated by the Monte Carlo simulation. It is believed that the model considered in this paper is a closer representation to reality than those reported earlier in the literature.

1. Introduction

The study of motion of ships or similar slender offshore structures in irregular seas has been performed over the years due to the fact that ship safety is a very expensive issue. Central to the study is the problem of predicting ship rolling motion due to random waves and associated first passage problem.

Owing to the strongly nonlinear nature of both the hydrodynamic damping and restoring moments, and the non-stationary random nature of excitation and response processes no closed-form solution to the governing equation of motion is possible and therefore approximate solution techniques have to be employed. Most of the available approximate solution techniques can only deal with small nonlinearities (see for example Refs. [1–4]). The model of nonlinear ship rolling motion in Refs. [1–4] is a nonlinear single degree-of-freedom

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(sdof) system under broad band stationary excitation. The damping in this model is of the linear-plus-quadratic type while the stiffness is assumed to be of linear-minus-cubic term. The later has the softening characteristics. This model appears to cater the essential features of ship rolling in irregular seas. The techniques employed in Refs. [1–4] are essentially the generalized stochastic averaging (GSA) with statistical or equivalent linearization [3] and the averaging procedure with assumption of the energy envelope of the single degree-of-freedom nonlinear system as a one-dimensional (1D) continuous Markov process [1,2,4]. While they are powerful analytical tools for dealing with such a problem the accuracy of generalized stochastic averaging depends, to a large extent, on the ratio, κ of bandwidth of the excitation process to that of the response process. For the generalized stochastic averaging to give accurate results the ratio κ has to be much greater than unity. In other words, the generalized stochastic averaging technique hinges on the assumption that damping is light. Moreover, small levels of excitation were assumed such that the response is approximately stationary.

The techniques of statistical nonlinearization (SNL) [5] and generalized stochastic averaging of energy envelope [6] have been employed to obtain the joint probability density function of displacement and velocity. In Ref. [5] the excitation process was assumed to be stationary white noise although its intensity and magnitudes of nonlinearities can be large, whereas in Ref. [6] the excitation process was broad band stationary.

Other approaches for nonlinear ship rolling in random seas can be found in Refs. [7–11]. In Ref. [7] the joint probability density function of nonlinear rolling motion was obtained with the non-Gaussian closure technique. Some researchers [8,9] have employed the Melnikov function approach that is popular in chaotic vibration studies of deterministic nonlinear systems.

The first-order approximation to the solution of the Fokker–Planck–Kolmogorov equation was presented in Ref. [10]. The second-order approximation solution appeared in Ref. [11].

With its proven accuracy, computational efficiency and the fact that it is applicable to systems having large nonlinearities and intense narrow band non-stationary random excitations, the generalized extended stochastic central difference (GESCD) method [12] is applied to investigate the response statistics of nonlinear ship rolling motion. The focus in the present studies is, however, on the capability of the proposed method rather than on specific system parameters associated with practical design consideration. It may be noted that high levels of damping are more typically encountered in practice [4]. The GESCD method is based on a combination of the extended stochastic central difference (ESCD) method [13] with statistical linearization (SL) technique, modified adaptive time scheme (MATS) [12], and time coordinate transformation (TCT) [14]. In this paper, the first passage probabilities based on the modified mean rate of various crossings, proposed earlier by Vanmarcke [15] for normal stationary random excitations and subsequently extended by To [16] for linear systems under non-stationary random excitations are also obtained. The approximate first passage probabilities in Ref. [16] has been successfully applied to discretized geometrically nonlinear plate structures under non-stationary random excitations [17]. This approach with due modification to the time step sizes of computing the first passage probabilities of nonlinear systems is applied in the present investigation. Monte Carlo simulation (MCS) results are also obtained for representative cases so that direct comparison can be made.

The contents of the remaining sections are as follows. Section 2 is concerned with the introduction and derivation of recursive expressions of the GESCD method for the response analysis of nonlinear ship rolling in narrow band stationary and non-stationary random seas. First passage probabilities of the nonlinear system are considered in Section 3. Responses computed by using the GESCD method and Monte Carlo simulation are presented in Section 4. Computed first passage probabilities of nonlinear ship rolling motion based on type-D crossings are also included. Conclusions are drawn and presented in Section 5.

2. Nonlinear ship rolling in narrow band non-stationary random seas

The model of nonlinear ship rolling in narrow band non-stationary random beam seas has the following governing equation of motion:

$$\ddot{y} + \alpha\dot{y} + \beta\dot{y}\left|\dot{y}\right| + \gamma y - \mu y^3 = f(t), \tag{1}$$

where y is the rolling angle measured in degrees; α, β, γ and μ are constant; and $f(t)$ is the narrow band non-stationary random excitation process, which is the solution of the filter to be defined later in the following.

Three aspects of difference between Eq. (1) and that by Roberts and associates [1–4] should be mentioned. First, the excitation process on the right-hand side of Eq. (1) is assumed to be narrow band non-stationary random while the corresponding term in Refs. [1–4] was assumed to be broad band stationary random or non-white process. On physical grounds, the excitation process considered in the present investigation is a more accurate representation since it has been reported that the excitation process is narrow band stationary random [18], for example. In the latter reference it was reported that simulation data of rolling responses of damage ships did not qualify to be stationary random. Second, the intensity of the excitation process on the right-hand side of Eq. (1) is not limited to a small value. The latter is a requirement in the solution considered in Refs. [1–4]. This requirement, in turn, confined the damping force in Refs. [1–4] to be small. On the other hand, the damping force considered in Eq. (1) can be large. In fact, high levels of damping are typical in practice [4]. Third, in Refs. [1–4] the energy envelope process is approximated as a 1D continuous Markov process, whereas in the present investigation no such approximation is assumed and therefore Eq. (1) in the present investigation is more general.

The narrow band random excitation process in Eq. (1) is governed by the following filter equation:

$$m_f\ddot{f} + c_f\dot{f} + k_f f = r(t) = e_f(t)w(t), \tag{2}$$

where m_f, c_f and k_f are the mass, damping and stiffness coefficients of the filter and e_f is a time-dependent deterministic modulating function. The zero-mean stationary Gaussian white noise process $w(t)$ has the spectral density S_0. Thus, the right-hand side of Eq. (2) is the non-stationary random excitation to the filter. Of course, if the deterministic modulating function is set to unity the input to the filter becomes a stationary random process. By changing the natural frequency, damping ratio of the filter and the spectral density of the Gaussian white noise excitation, a variety of narrow band random processes, in the time domain, with different properties such as central frequencies and bandwidths can be obtained [12,13].

2.1. Discretized equations and application of statistical linearization

Eq. (1) cannot be solved in closed form and therefore an approximated solution technique is applied. The first stage in the approximation is the discretization in the time domain such that the equations for the filter and ship rolling motion are, respectively,

$$m_f\ddot{f}(s) + c_f\dot{f}(s) + k_f f(s) = e_f(s)w(s) \tag{3}$$

and

$$\ddot{y}(s) + h(y(s), \dot{y}(s))\dot{y}(s) + g(y(s))y(s) = f(s), \tag{4}$$

in which $h(.)$ and $g(.)$ denote, respectively, the nonlinear damping and nonlinear stiffness coefficients in Eq. (1), while s represents the number of time step in the time domain such that $t_{s+1} - t_s = \Delta t$.

The difference equation in Eq. (4) is nonlinear. However, at every time step the statistical linearization technique can be applied. It may be appropriate to note that while in the statistical linearization technique the response process at every time step is assumed to be Gaussian the response for the entire time duration is non-Gaussian. This is different from the conventional statistical linearization technique in which an equivalent system is determined for the entire time range [19]. After the application of the statistical linearization technique to Eq. (4) at every time step and some algebraic manipulation one can show that the equivalent equation becomes

$$\ddot{y}(s) + 2\zeta_e(s)\omega_e(s)\dot{y}(s) + \omega_e^2(s)y(s) = f(s), \tag{5}$$

where

$$2\zeta_e(s)\omega_e(s) = \alpha + \sqrt{\frac{8}{\pi}}\beta\sigma_{\dot{y}}(s) \tag{6}$$

and

$$\omega_e^2(s) = \gamma - 3\mu\sigma_y^2(s), \tag{7}$$

in which $\zeta_e(s)$ and $\omega_e(s)$ are the equivalent damping ratio and equivalent natural frequency of the system at time step t_s, whereas $\sigma_y(s)$ and $\sigma_{\dot{y}}(s)$ are the standard deviations of angular displacement y and angular velocity $\dot{y}$ at time step t_s.

2.2. Recursive responses by generalized extended stochastic central difference method

Having applied the statistical linearization technique at every time step, the next stage in the GESCD method [12] is to obtain the recursive variance expressions for angular displacement. By applying Eq. (5) and the steps in Ref. [13], it can be shown that the recursive variance of angular displacement is

$$\begin{aligned} R_y(s+1) = {} & N_{2y}^2 R_y(s) + N_{3y}^2 R_y(s-1) + (\Delta t)^4 N_{1y}^2 R_f(s) + 2N_{2y} D_y(s) N_{3y} + 2(\Delta t)^2 N_{2y} G(s) N_{1y} \\ & + 2(\Delta t)^2 N_{3y} H(s) N_{1y}, \end{aligned} \tag{8}$$

where $f_s = f(s)$, $y_s = y(s)$, $G(s) = \langle y_s f_s \rangle$, $H(s) = \langle y_{s-1} f_s \rangle$, $R_f(s) = \langle f_s^2 \rangle$, $R_y(s) = \langle y_s^2 \rangle$,

$$\begin{aligned} D_y(s) &= \langle y_s y_{s-1} \rangle \\ &= N_{2y} R_y(s-1) + N_{3y} D_y(s-1) + (\Delta t)^2 N_{1y} G(s-1), \end{aligned} \tag{9}$$

$$N_{1y} = \left[1 + \frac{1}{2}(\Delta t) 2\zeta_e(s)\omega_e(s)\right]^{-1}, \quad N_{2y} = N_{1y}\left[2 - (\Delta t)^2 \omega_e^2(s)\right], \tag{10a,b}$$

$$N_{3y} = N_{1y}[(\Delta t)\zeta_e(s)\omega_e(s) - 1] \tag{10c}$$

and $R_f(s)$ is the recursive variance of the response of the filter while the angular brackets denote an ensemble average or mathematical expectation of the enclosing quantity. By applying the stochastic central difference (SCD) method [20] to Eq. (3), one can show that the recursive variance of filter response is

$$R_f(s+1) = N_{2f}^2 R_f(s) + N_{3f}^2 R_f(s-1) + (\Delta t)^4 N_{1f}^2 R_w(s) + 2N_{2f} D_f(s) N_{3f}, \tag{11}$$

where the variance of uniformly modulating discrete white noise process is $R_w(s) = 2\pi S_0 e_f^2(s)$ and

$$D_f(s) = \langle f_s f_{s-1} \rangle = N_{2f} R_f(s-1) + N_{3f} D_f(s-1), \tag{12}$$

$$N_{1f} = \left[m_f + \frac{1}{2}(\Delta t) c_f\right]^{-1}, \quad N_{2f} = N_{1f}\left[2m_f - (\Delta t)^2 k_f\right], \tag{13a,b}$$

$$N_{3f} = N_{1f}\left[\frac{1}{2}(\Delta t) c_f - m_f\right]. \tag{13c}$$

In Eq. (8) the recursive expressions $G(s)$ and $H(s)$ that carry the effects of the narrow band random process can be obtained by the ESCD method [13] as

$$G(s) = (\Delta t)^2 N_{1y} D_f(s) + N_{2y} H(s) + N_{3y} H(s-1) N_{2f} + N_{3y} G(s-2) N_{3f} \tag{14}$$

and

$$H(s) = G(s-1) N_{2f} + (\Delta t)^2 N_{1y} R_f(s-2) N_{3f} + N_{2y} G(s-2) N_{3f} + N_{3y} H(s-2) N_{3f}. \tag{15}$$

For convenience of reference, Eqs. (8)–(15) are collectively referred as the GESCD method. Of course, Eqs. (6) and (7) contain the standard deviations of angular displacement and angular velocity, $\sigma_y(s)$ and $\sigma_{\dot{y}}(s)$. The variance of angular displacement is in fact, $\sigma_y^2(s) = R_y(s)$. Therefore, in the present problem the recursive variance of angular velocity has to be derived independently. To this end, one may apply the deterministic central difference method so that the angular velocity at t_s becomes

$$\dot{y}(s) = \frac{1}{\Delta t}[y(s) - y(s-1)]. \tag{16}$$

By multiplying Eq. (16) by itself and taking the ensemble averages on both sides of the resulting equation, it reduces to

$$\sigma_{\dot{y}}^2(s) = \left\langle \dot{y}^2(s) \right\rangle = \frac{1}{(\Delta t)^2}\left[\langle y^2(s)\rangle + \langle y^2(s-1)\rangle - 2\langle y(s)y(s-1)\rangle\right]. \tag{17}$$

This completes the derivation of the recursive expressions in the GESCD method with particular reference to ship rolling in narrow band stationary and non-stationary random beam seas. As pointed out in Ref. [12], the time step sizes for the filter and system are different since their natural frequencies at every time step are not identical in general. Consequently, at every time step an output from the filter cannot be directly applied as an input to the system of interest. To resolve this problem the time step size of the output from the filter is interpolated. This strategy of dealing with different time step sizes between the filter and the system has been proved to be efficient and accurate [12,13]. It has been demonstrated that without such an interpolation computational instability can occur [12]. In addition, since the system is nonlinear and therefore the modified adaptive time scheme (MATS) together with the time coordinate transformation (TCT) for stiff system are included in the GESCD method.

Finally, as mentioned in Ref. [13], by applying different envelope functions $e_f(s)$, constant $2\pi S_0$ which is I in Refs. [12,13], the natural frequencies of the filters and the ratios of damping to mass, a variety of different shapes, spectral densities, center frequencies and bandwidths of the narrow band random processes from the filters can be obtained. This is a unique and efficient feature of the ESCD method for linear systems and GESCD method for nonlinear systems.

3. First passage probabilities of nonlinear systems

With the time-dependent recursive mean squares or variances of angular displacements and velocities obtained, the first passage problem can now be considered. Approximate first passage probabilities based on the modified mean rate of various crossings proposed earlier by the first author [16,17] can be computed. In the latter work the trapezoidal rule with uniform time step size was employed to evaluate the first passage probabilities. However, in the present investigation because of the large nonlinearities of the system and the fact that the natural frequency at every time step is different from one time step to another, the first passage probabilities are evaluated by the trapezoidal rule with variable time step sizes.

In the present investigation only the distribution of the first passage probabilities for narrow band and wide band non-stationary random processes based on type-D crossings is included in this section. Other types of barriers can be found in Ref. [16]. The first passage probability for type-D barrier based on Poisson process assumption is [15]

$$L_D(t) = \mathrm{e}^{-2\int_0^t v_b(u)\,\mathrm{d}u}, \tag{18}$$

where

$$v_b(u) = v_b = v_0\,\mathrm{e}^{-b/2\sigma_u^2}, \tag{19}$$

in which b is the barrier level and v_0 is the zero crossing rate. Thus, $L_D(t)$ in Eq. (18) is the probability that the time of first passage of the level b by $|u(t)|$ is greater than t. It is also the probability that the absolute value of the process remains below b at all times in the interval $[0, t]$. Eq. (18) has been numerically evaluated by the trapezoidal rule using constant time step size in Refs. [16,17] as

$$L_D(s) = \mathrm{e}^{-(\Delta t)[\alpha_D(1)+\alpha_D(2)+\cdots+\alpha_D(s)]}, \tag{20}$$

where $L_D(s)$ is $L_D(t)$ evaluated at time t_s and $\alpha_D(s)$ is $\alpha_D(t) = 2v_b(t)$ at t_s.

For nonlinear systems the natural frequencies, and therefore the time step sizes, at every time step are different such that Eq. (20) becomes

$$L_D(s) = e^{-\left[(\Delta t)_1 \alpha_D(1) + (\Delta t)_2 \alpha_D(2) + \cdots + (\Delta t)_s \alpha_D(s)\right]}, \tag{21}$$

in which $(\Delta t)_s$ is the time step size at time step t_s and is evaluated in accordance with Eq. (10) of Ref. [13].

Expressions similar to Eq. (21) for other types of barriers can be derived but for brevity they are not considered here.

4. Computed recursive nonlinear responses and first passage probabilities

Three cases are considered in the following sub-sections. In the first case, results are obtained for narrow band non-stationary random excitations. In the second case, comparisons are made of responses of narrow band stationary random responses to wide band stationary random response. The third case in Subsection 4.3 is concerned with comparison of narrow band non-stationary random responses to wide band non-stationary random responses. In the first case, results by the Monte Carlo simulation technique are included for direct comparison. Furthermore, in Subsection 4.4 first passage probabilities based on type-D crossings are presented. Remarks are included in Subsection 4.5.

For identification with given system parameters in Eq. (1) and known definitions of linear single degree-of-freedom oscillator, Eqs. (6) and (7) are re-written as in the following manner:

$$2\zeta_e(s)\omega_e(s) = 2\zeta_s\left[1 + \sqrt{\frac{8}{\pi}}\eta\sigma_{\dot{y}}(s)\right] \tag{22}$$

and

$$\omega_e^2(s) = \omega_s^2\left[1 - 3\varepsilon\sigma_y^2(s)\right], \tag{23}$$

where ζ_s and ω_s are the damping ratio and natural frequency of the associated linear system. Thus, the system parameters in Eq. (1) are

$$\alpha = 2\zeta_s, \quad \beta = 2\zeta_s\eta, \quad \gamma = \omega_s^2, \quad \mu = \varepsilon\omega_s^2.$$

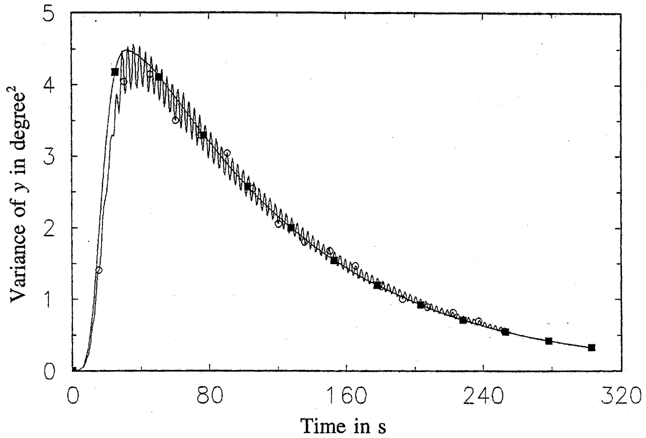

Fig. 1. Response variance of rolling ship model under narrow band non-stationary random excitation with $\omega_f = 1.0$ rad/s, $\zeta_f = 0.01$, $\omega_s = 1.0$ rad/s, $\zeta_s = 0.0125$, $\eta = 0.04$, $\varepsilon = 5.0$. Monte Carlo simulation (○), and GESCD (■).

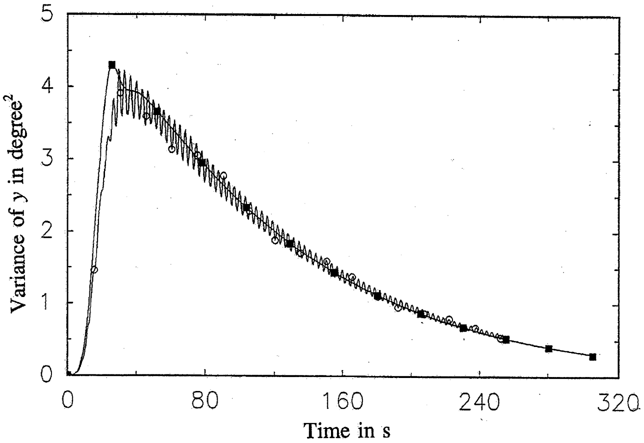

Fig. 2. Response variance of rolling ship model under narrow band non-stationary random excitation with $\omega_f = 1.0\,\text{rad/s}$, $\zeta_f = 0.01$, $\omega_s = 1.0\,\text{rad/s}$, $\zeta_s = 0.0125$, $\eta = 0.4$, $\varepsilon = 8.0$. Monte Carlo simulation (○), and GESCD (■).

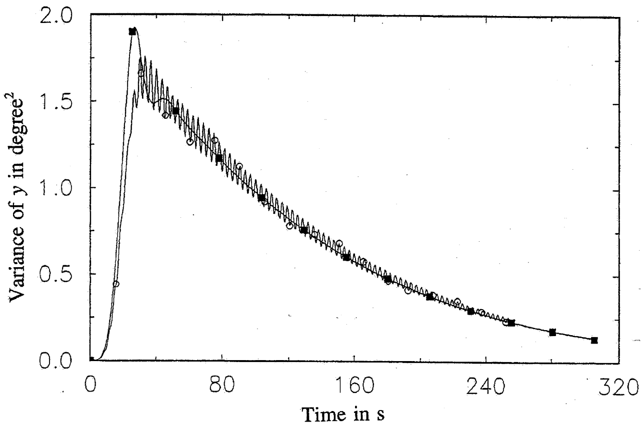

Fig. 3. Response variance of rolling ship model under narrow band non-stationary random excitation with $\omega_f = 1.0\,\text{rad/s}$, $\zeta_f = 0.01$, $\omega_s = 1.0\,\text{rad/s}$, $\zeta_s = 0.0125$, $\eta = 0.4$, $\varepsilon = 5.0$. Monte Carlo simulation (○), and GESCD (■).

Table 1
Parameters of nonlinear rolling ship model

Figure number	1	2	3
m_s (kg)	1.0	1.0	2.0
ω_s (rad/s)	1.0	1.0	1.0
ζ_s (%)	1.25	1.25	1.25
η	0.04	0.4	0.4
ε	5.0	8.0	5.0
$(\sigma_y^2)_{\max}$	26,633	26,633	6658

4.1. System under narrow band non-stationary random excitation

A uniformly modulated zero mean Gaussian white noise is input to the filter. The deterministic uniformly modulated function is given by

$$e_f(t) = 4.0\left(\mathrm{e}^{-0.05t} - \mathrm{e}^{-0.1t}\right). \tag{24}$$

Monte Carlo simulation results based on Eqs. (1) and (2) together with those evaluated by the GESCD method are included in Figs. 1–3. It may be appropriate to note that in the Monte Carlo simulation results the time step sizes employed in the numerical integration scheme, the Runge–Kutta fourth-order algorithm are constant. They are different from those selected in the GESCD method. In the latter method the time step sizes for the filter and system were computed by applying Eq. (10) of Ref. [13] in addition to the interpolation scheme mentioned in Section 2. The spectral density of the excitation to the filter is $S_0 = 0.01$ unit and the filter mass $m_f = 1.0$ kg. The filter natural frequency is 1.0 rad/s, and filter damping ratio is 0.01. The properties of the rolling ship are listed in Table 1 in which the subscript s denotes the rolling ship. It should be noted that in the Monte Carlo simulation every solution was evaluated with 200 realizations, each of which has 25,600 points.

In Table 1 the term $(\sigma_y^2)_{\max}$ is the maximum variance of displacement response for the corresponding linear system. It is used to normalize the nonlinear stiffness coefficient. One typical feature shown by the foregoing figures is that the system responses attenuate rapidly. This is because the nonlinear damping increases with increasing angular velocity standard deviation. The figures show that the schemes implemented in the GESCD method such as the modified adaptive time scheme, input interpolation, expression of angular velocity variance are correct since results by the GESCD method compared very well with those applying the Monte Carlo simulation. This, in turn, indicates that the GESCD method is an excellent alternative to various approximate techniques available in the literature for solution of nonlinear ship rolling motion in narrow band stationary and non-stationary random seas.

Table 2
Parameters for stationary response cases

White noise	Narrow band	
$S_0 = 0.01$	$2\pi S_0 = 0.0628$ $\omega_f = 1.0$ rad/s	$\zeta_f = 1.0\%$ $\zeta_f = 5.0\%$ $\zeta_f = 10.0\%$
$\varepsilon = 0.01$, $\eta = 0.02$, $\zeta_s = 1.0\%$, $\omega_s = 1.0$ rad/s, $\Delta t = 0.83$ s		
$S_0 = 0.01$	$2\pi S_0 = 0.0628$ $\omega_f = 1.0$ rad/s	$\zeta_f = 1.0\%$ $\zeta_f = 5.0\%$ $\zeta_f = 10.0\%$ $\zeta_f = 100.0\%$
$\varepsilon = 0.02$, $\eta = 0.5$, $\zeta_s = 1.0\%$, $\omega_s = 1.0$ rad/s, $\Delta t = 0.83$ s		

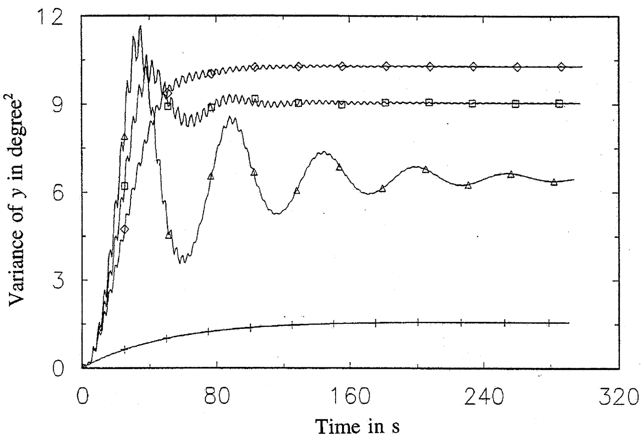

Fig. 4. Effect of bandwidth on variance of ship motion with narrow band stationary random excitation with $\omega_f = 1.0$ rad/s, $\omega_s = 1.0$ rad/s, $\zeta_s = 0.01$, $\eta = \varepsilon = 0.01$. $\zeta_f = 0.01$ (△), $\zeta_f = 0.05$ (□), $\zeta_f = 0.10$ (◇), and white noise (+).

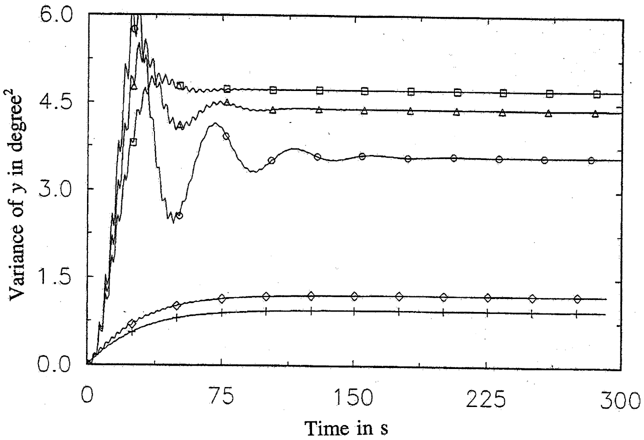

Fig. 5. Effect of bandwidth on variance of ship motion with narrow band stationary random excitation with $\omega_f = 1.0$ rad/s, $\omega_s = 1.0$ rad/s, $\zeta_s = 0.01$, $\eta = 0.5$, $\varepsilon = 0.02$. $\zeta_f = 0.01$ (○), $\zeta_f = 0.05$ (△), $\zeta_f = 0.10$ (□), $\zeta_f = 1.00$ (◇), and white noise (+).

4.2. Comparison of narrow and wide band stationary random responses

The computed results in this case are used to show the effect of bandwidth of narrow band process on stationary random response. A parametric study is performed on the system with various narrow band stationary random excitations as well as different center frequencies of the filter. The responses are compared with those of the system subject to zero mean Gaussian white noise excitations. The spectral density of the white noise inputs to the system is $S_0 = 0.01$ unit, thus, $R_w(s) = 0.0628$. That is, the constant of the narrow band input to the system is $2\pi S_0 = 0.0628$ which is defined in Eq. (11). Two examples are considered in this subsection. The pertinent parameters are listed in Table 2. Computed results are included in Figs. 4 and 5.

From the results in Figs. 4 and 5, several observations should be noted. Firstly, all responses to narrow band random excitations have overshoots while for the white noise and broad band processes it does not have overshoot. Secondly, the narrower the bandwidth of the narrow band excitation is, the longer it takes for the system response to reach its stationary value. Thirdly, for narrow band stationary random excitation, the magnitude of response increases with increasing bandwidth when the latter is relatively small. However, when the bandwidth becomes large the magnitude of response decreases with increasing bandwidth. Fourthly, similar to the results obtained for a Duffing oscillator studied in Ref. [12], the increase of nonlinear coefficient reduces the amplitude of system response.

4.3. Comparison of non-stationary random responses

In this sub-section, narrow band non-stationary random excitations are input to the rolling ship model and its responses are compared with those of the model under uniformly modulated Gaussian white noise excitation. The pertinent data for this case are presented in Table 3. The deterministic modulating function is defined by Eq. (22). Computed results are included in Figs. 6 and 7.

With reference to the results in Figs. 6 and 7, the following conclusions can be drawn. First, in contrast to their stationary counterparts, the responses to narrow band non-stationary random excitation increase with the decreasing bandwidth. Second, the bandwidth effect on the amplitudes of responses to narrow band excitation is more significant than that in the stationary cases. In other word, the value of the amplitude difference of responses to narrow band non-stationary excitations is larger than that of responses to narrow band stationary excitation. Of course, in this comparison the damping ratios of the filter in the non-stationary excitation cases are the same as those for the stationary random excitation ones. Third, similar to the linear cases, amplitudes of responses decrease with increasing natural frequency of the system.

Table 3
Parameters for nonstationary response cases

Wide band	Narrow band	
$S_0 = 0.01$	$2\pi S_0 = 0.01$	$\zeta_f = 5.0\%$
	$\omega_f = 1.0\,\text{rad/s}$	$\zeta_f = 10.0\%$
		$\zeta_f = 100.0\%$
$\omega_s = 1.0\,\text{rad/s}$, $\varepsilon = 0.02$, $\eta = 0.5$, $\zeta_s = 1.0\%$, $m_s = 1.0\,\text{kg}$, $\Delta t = 0.83\,\text{s}$ for corresponding linear model		
$S_0 = 0.01$	$2\pi S_0 = 0.01$	$\zeta_f = 1.0\%$
	$\omega_f = 3.0\,\text{rad/s}$	$\zeta_f = 5.0\%$
		$\zeta_f = 10.0\%$
		$\zeta_f = 100.0\%$
$\omega_s = 3.0\,\text{rad/s}$, $\varepsilon = 0.02$, $\eta = 0.5$, $\zeta_s = 1.0\%$, $m_s = 1.0\,\text{kg}$, $\Delta t = 0.48683\,\text{s}$ for corresponding linear model		

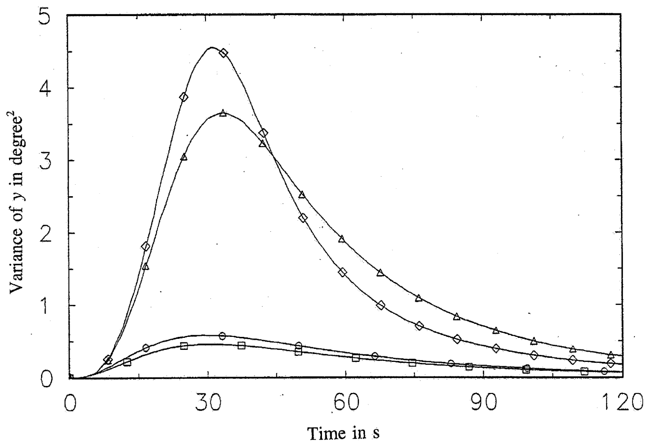

Fig. 6. Effect of bandwidth on variance of ship motion with narrow band non-stationary random excitation with $\omega_f = 1.0\,\text{rad/s}$, $\omega_s = 1.0\,\text{rad/s}$, $\zeta_s = 0.01$, $\eta = \varepsilon = 0.01$. $\zeta_f = 1.00$ (○), $\zeta_f = 0.10$ (△), $\zeta_f = 0.05$ (◇), and white noise (□).

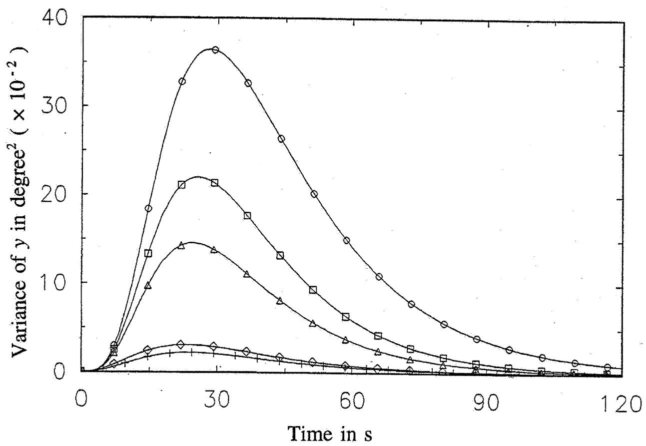

Fig. 7. Effect of bandwidth on variance of ship motion with narrow band non-stationary random excitation with $\omega_f = 1.0\,\text{rad/s}$, $\omega_s = 1.0\,\text{rad/s}$, $\zeta_s = 0.01$, $\eta = 0.5$, $\varepsilon = 0.02$. $\zeta_f = 0.01$ (○), $\zeta_f = 0.05$ (□), $\zeta_f = 0.10$ (△), $\zeta_f = 1.00$ (+), and white noise (◇).

4.4. First passage probabilities based on type-D crossings

Having studied the difference between the nonlinear ship rolling in wide band and narrow band random seas, and the efficiency as well as accuracy of responses evaluated by the GESCD method, it is logical and natural to investigate the differences, if any, between the associated first passage probabilities. In this respect, Eq. (21) is applied.

Some representative results based on the the so-called D-crossing are presented in Fig. 8 in which GESCD denotes those computed by the GESCD method, while Monte Carlo simulation denotes those obtained by the Monte Carlo simulation for Eq. (1) in which $\alpha = \beta = 0.025$, $\gamma = 1.0$ and $\mu = 0.0045$ whereas the spectral density $S_0 = 1.0$ unit. The deterministic modulating function of the wide band excitation process applied to the filter is defined in Eq. (24). The barrier level in this case is $b = 2\left[(\sigma_y^2)_{\max}\right]^{1/2}$. Fig. 9 includes results based on the D-crossing and evaluated by the GESCD method. System parameters for this case are those applied for

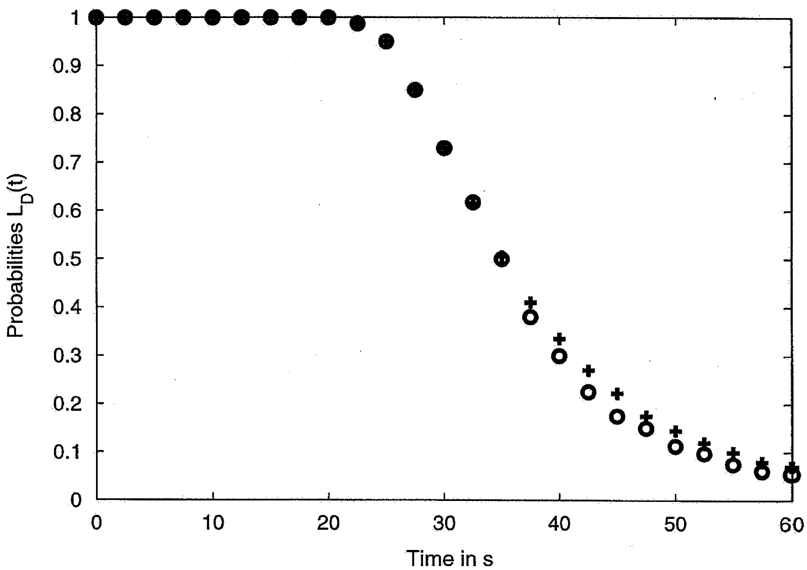

Fig. 8. Probabilities of D-crossing of nonlinear ship rolling with $b = 2\left[(\sigma_y^2)_{\max}\right]^{1/2}$. Monte Carlo simulation (+), and GESCD (○).

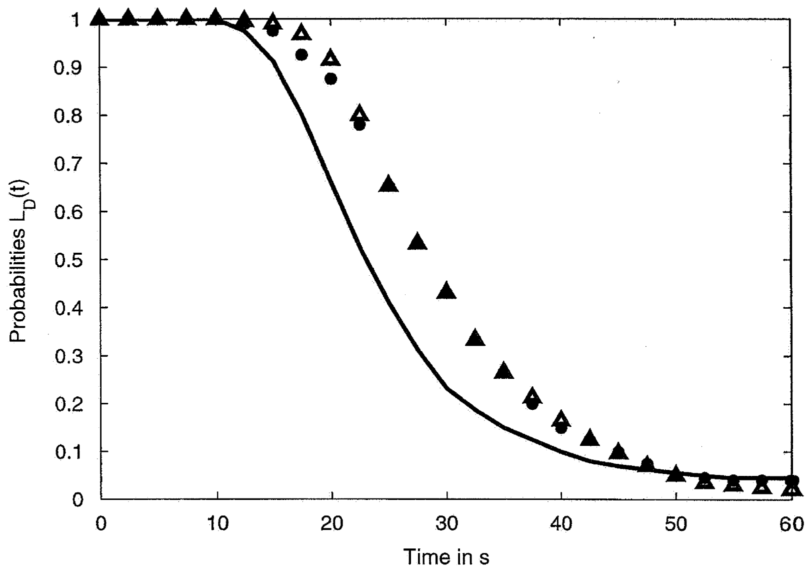

Fig. 9. Probabilities of D-crossing of nonlinear ship rolling with $b = \left[(\sigma_y^2)_{\max}\right]^{1/2}$. Wide band non-stationary random (__), narrow band non-stationary random (△), and narrow band stationary random (●).

the results in Fig. 8, except that the excitation to the system is narrow band random with the input to the filter having a modulating function defined in Eq. (24) and spectral density $S_0 = 1.0$ unit. The parameters for the filter are those presented in Section 4.1. That is, $\omega_f = 1.0\,\mathrm{rad/s}$, $\zeta_f = 0.01$ and $m_f = 1.0\,\mathrm{kg}$. The barrier level in this case is $b = \left[(\sigma_y^2)_{\max}\right]^{1/2}$. With reference to the computed results presented in Fig. 8, one observes that the GESCD method is very accurate compared with the Monte Carlo simulation data. However, the GESCD method is far more efficient in that the ratio of computational time applying the Monte Carlo simulation to that by the GESCD method is over 200 on average. In regard to the results in Fig. 9, one can observe that the first passage probability of the model with narrow band stationary or non-stationary random excitation is significantly different from that with wide band non-stationary excitation. On the other hand, there is insignificant difference between results for the narrow band stationary and non-stationary random excitations. This again emphasizes the importance of treating the random waves as narrow band random processes.

4.5. Remarks

In the foregoing it has been demonstrated that results computed by the GESCD method have excellent agreement with those obtained by the Monte Carlo simulation. It is understood that the system parameters applied above do not necessarily represent those of a particular ship model. The parameters were chosen to demonstrate the capability of the GESCD method for relatively large nonlinearities and intensive excitations.

5. Conclusions

Responses and first passage probabilities based on type-D crossings of nonlinear ship rolling motion have been studied and presented in this paper. The emphasis of the studies was on the development and presentation of the GESCD method for analysis of nonlinear ship rolling rather than on effect and implication of response analysis of specific ship design.

Two important and new features of the approach introduced in the foregoing are: (a) the wave motion is considered as a narrow band non-stationary random process, and (b) the nonlinearities and intensity of excitation process are large and therefore the present model is different from those available in the literature. Unlike Refs. [1–4], for example, in which the energy envelope process was approximated as a 1D continuous Markov process, the present approach does not require such an approximation. Thus, the approach introduced in this paper is mathematically simpler and the nonlinear ship rolling motion model is physically more general.

Explicit recursive expressions of the GESCD method for the nonlinear ship rolling motion model have been derived. First passage probabilities based on type-D crossings of nonlinear systems has been approximated with the trapezoidal rule. Computed results by the Monte Carlo simulation are also presented for direct comparison. It is observed that results by the GESCD method and Monte Carlo simulation are in excellent agreement. The GESCD method is very efficient compared with the Monte Carlo simulation. Computed results also indicate the importance of treating the excitation process as narrow band random. It may be appropriate to note that as the method proposed is for response analysis of ship rolling motion with large nonlinearities and intensive stationary and non-stationary random excitations, comparisons were made only between results of the proposed method and those of Monte Carlo simulation, since other available approaches are only applicable to systems with small nonlinearities and weak stationary random excitations.

For the particular system parameters considered in the present studies, it is also observed that (a) the amplitudes of responses of the resonant systems are much larger than those of their corresponding non-resonant counterparts which is as expected; (b) multiple peaks of variance occur when the natural frequency of the system is near the center frequency of the narrow band non-stationary random excitation; (c) the difference between responses to narrow band and to white noise excitation decreases with increasing frequency; (d) the difference between responses to narrow band and to white noise excitation decreases with increasing of stiffness nonlinearity; (e) the number of peaks of responses to narrow band excitation increases with increasing nonlinear coefficient; and (f) magnitudes of peaks of responses reduce with increasing nonlinear stiffness coefficient.

Finally, it should be stated that with the proposed approach limiting or capsizing moments of typical ships can be obtained and examined. These constitute the second phase of the investigation and shall be reported in due course.

References

[1] J.B. Roberts, A stochastic theory for nonlinear ship rolling in irregular seas, *Journal of Ship Research* 26 (1982) 229–245.
[2] J.B. Roberts, N.M.C. Dacunha, Roll motion of a ship in random beam waves: comparison between theory and experiment, *Journal of Ship Research* 29 (1985) 112–126.
[3] J.B. Roberts, J.F. Dunne, A. Debonos, Stochastic estimation methods for nonlinear roll motion, *Probabilistic Engineering Mechanics* 9 (1994) 83–93.
[4] J.B. Roberts, M. Vasta, Markov modelling and stochastic identification for nonlinear ship rolling in random waves, *Philosophical Transactions of the Royal Society of London A* 358 (2000) 1917–1941.
[5] C.W.S. To, A statistical non-linearization technique in structural dynamics, *Journal of Sound and Vibration* 161 (1993) 543–548.
[6] C.W.S. To, A general theory of the stochastic averaging method of energy envelope, *Journal of Sound and Vibration* 212 (1998) 165–172.
[7] M.R. Haddara, Y. Zhang, On the joint probability density function of nonlinear rolling motion, *Journal of Sound and Vibration* 169 (1994) 562–569.
[8] S.R. Hsieh, A.W. Troesch, S.W. Shaw, A nonlinear probabilistic method for predicting vessel capsizing in random beam seas, *Proceedings of the Royal Society of London A* 446 (1994) 195–211.
[9] C.B. Jiang, A.W. Troesch, S.W. Shaw, Highly nonlinear rolling motion of biased ships in random beam seas, *Journal of Ship Research* 40 (1996) 125–135.
[10] N.K. Moshchuk, R.A. Ibrahim, R.Z. Khasminiskii, P.L. Chow, Asymptotic expansion of ship capsizing in random sea waves. 1: First-order approximation, *International Journal of Non-Linear Mechanics* 30 (1995) 727–740.
[11] N.K. Moshchuk, R.A. Ibrahim, R.Z. Khasminiskii, P.L. Chow, Asymptotic expansion of ship capsizing in random sea waves. 2: Second-order approximation, *International Journal of Non-Linear Mechanics* 30 (1995) 741–757.
[12] Z. Chen, C.W.S. To, Responses of discretized systems under narrow band nonstationary random excitations, part II: nonlinear problems, *Journal of Sound and Vibration* 287 (2005) 459–479.
[13] Z. Chen, C.W.S. To, Responses of discretized systems under narrow band nonstationary random excitations, part I: linear problems, *Journal of Sound and Vibration* 287 (2005) 433–458.
[14] C.W.S. To, M.L. Liu, Random responses of discretized beams and plates by the stochastic central difference method with time co-ordinate transformation, *Computers and Structures* 53 (1994) 727–738.
[15] E.H. Vanmarcke, On the distribution of the first passage time for normal stationary random processes, *Journal of Applied Mechanics* 42 (1975) 215–220.
[16] C.W.S. To, Distribution of the first passage time of mast antenna structures to non-stationary random excitation, *Journal of Sound and Vibration* 108 (1986) 11–23.
[17] C.W.S. To, First passage time of discretized plates with geometrical nonlinearity, *Computers and Structures* 24 (1986) 893–900.
[18] P. Valanto, *Time-dependent survival probability of a damaged passenger ship, The Hamburg Ship Model Basin*, Vol.1, HSVA Report Number CFD 05/2002, 2002.
[19] T.K. Caughey, Equivalent linearization techniques, *The Journal of the Acoustical Society of America* 35 (1963) 1706–1711.
[20] C.W.S. To, The stochastic central difference method in structural dynamics, *Computers and Structures* 23 (1986) 813–818.